Utilization of Queuing Mathematical Theory in the Effective Management of Outpatient Services in a University Teaching Hospital in South - South Geo – Political Zone, Nigeria

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This work is original and was carried out in collaboration between all authors mentioned above. Author Adie, J.A wrote the protocol and the first draft of the manuscript. Author Beshel, I.A. designed the study and performed the statistical analysis, Author Nwani, O.B.J. managed the analyses of the study and the literature searches. All authors read and approved the final manuscript.

ABSTRACT

This study examine the application of mathematical theory in the effective Management of outpatient services in a Federal University Teaching Hospital in South – South Geo – Political zone, Nigeria. The researchers prepared the ground for the study by identifying the research problem, the study objectives and research questions. Three hypothesis were framed for the study and a well-designed questionnaire was prepared and three hundred and twenty eight copies (328) were distributed to patients during outpatient clinic session for completion. The completed questionnaire were returned and data extracted for analysis. Percentage was used to compare the responses, using structured questionnaire. The result of the study revealed that: queuing of patients enhanced efficient service provision at the outpatient clinic of the Teaching Hospital; application of queuing mathematical theory improve patients' waiting time and queuing mathematical theory can serve as a suitable model for managing congestion at the outpatient clinic of the Teaching Hospital. Recommendations were proffered for implementation.

Key words: *Utilization, Queuing, Mathematical theory, Effective, Management, Outpatient Services.*

INTRODUCTION

The present economic scenario has created the need for decision – makers in hospitals to be prudent and efficient in the management of hospital resources for better results. To achieve this with high level of success, there is need to direct and maintain efficient control over those professionals responsible for manipulating the tools that allow them to analyze, program, plan, prioritize and, in general, decide on the best way to manage the available resources. Queueing mathematical theory is one of those options that can be applied to resolve such problems in health care organizations to ensure efficiency and patients' satisfaction (Benneyan, 1997).

Queuing mathematical theory is the study of all kinds of lines where there is crowd. Queuing can be applied in institutions like; hospitals, banks, supermarket checkouts, petrol filling stations, the lines at toll booths, and crowded areas where people wait for services. Queuing mathematical theory is used to analyze the congestions and delays in organizations where services are delivered and the resources available to provide the services are scare. It is particularly very useful in health care organizations, at the outpatient clinics of the hospital where the demand for medical services is greater than service provision. The theory can also be used to examine every component of services provision

including patients' waiting time, beginning from the arrival process when the services is offered till the client exit or leaves the health care organization (Adeleke, Adebiyi, and Akinyemi, 2009).

Green (2010) stressed that queuing theory can be used to estimate the level of services provided to the patients, determine the average waiting time and the number of patients queued, the capacity used, and the probability that the patient needs to wait. Thus, the waiting time to receive attention by the service provider becomes the key element in measuring the quality of the service. This theory is often applied to classify patients and assign doctors to them. The theory also works in real time, requiring heuristic methods to obtain a solution to a problem. Overcrowding in outpatient departments of most hospitals usually create problems and affects the ability to provide health services within a reasonable period of time. When the number of patients who present in outpatient department is growing and the department's ability to assist patients with acute complains is constant. Delays in such situations may cause dramatic outcomes for patients in outpatient Department in terms of patients flow in relation to the available resources. Outpatient Department in most Teaching hospitals can be regarded as a network of queues and different types of servers where patients arrive, wait for a service, get a result and then go home or they are admitted to a hospital ward. The waiting threads are effective tools to support management (Ajayi, 2002).

The real life application of queuing mathematical theory enhances faster services, improving traffic flow in the clinics and reduce patients' waiting time. It has the ability to accommodate random variation in patients' arrival time and helps the system focused on the well-being and life of the clients/ patients (Nosek, and Wilson, 2001).

STATEMENT OF THE PROBLEM

One of the most frequent problems confronting outpatient department of every hospital where there is overcrowding is the waiting time of the patient before services are provided (queue). This has become the most frustrating issue in health care delivery system. Waiting time for effective care has been considered a service problem. This is because it acts as a barrier to efficient patient flow at the outpatient unit and also hinder clients from coming to the hospital for consultation when sick.

Olorunsola, Adeleke and Ogunlade (2014) noted that, because of the growing tendency for specialized services, patients tend to visit the teaching hospitals rather than go to private clinics and other forms of health services provision. Thus, there is tendency for continuing increase in the number of patients visiting the outpatient clinics of the specialist hospitals for health services. This has resulted in patients arriving during peak to wait until it is their turn to be served. Other issues arising from poor organization of health services, which may be due to inefficiency, lack of coordination among those involve in service delivery, poor planning and inadequate human/ materials resources which often create bottle neck in services delivery at the outpatient clinics of the hospital.

OBJECTIVE OF THE STUDY

The objective of this study is to examine the utilization of Queuing mathematical theory in the effective management of outpatient services in a Teaching Hospital, in South – South Geo political zone of Nigeria. The specific objectives are as follows:

i. To ascertain whether queuing of patients enhanced efficient service provision at the outpatient clinic of the Teaching Hospital.
ii. To investigate whether application of queuing mathematical theory improve patients' waiting time at the outpatient clinics of the Teaching Hospital.
iii. To determine whether queuing mathematical theory could serve as a suitable model for managing congestion at the outpatient clinic of the Teaching Hospital.

RESEARCH QUESTIONS

The following research questions were framed for the purpose of the study.

i. Does Queuing of patients enhanced efficient service provision at the outpatient clinic of the Teaching Hospital?
ii. Does application of queuing mathematical theory improve patients' waiting time at the outpatient clinics of the Teaching Hospital?
iii. Does queuing mathematical theory ensure decongestion of patients at the outpatient clinic of the Teaching Hospital?

CONCEPTUAL ANALYSIS

The proponent of queueing mathematical theory was Agner Krarup Erlang, a Danish Engineer who worked for the Copenhagen Telephone Exchange. He first published a paper on what is now called queueing theory in 1909, when he created models to describe the Copenhagen telephone exchange. He modeled the number of telephone calls arriving at an exchange by a Poisson process and used it to solve the M/D/1 queue in 1917 and M/D/k model in 1920. Since then, many theorems in queueing theory have been proved by reducing queues to mathematical systems known as Markov chains, first described by Andrey Markov in his 1906 paper (Hill, and Joonas, 2005).

Queueing mathematical theory is considered to be the branch of operations research that was used to study waiting lines or queues. It is often used when making business decisions about the resources needed to provide a service. The theory can also be applied to the waiting time of out-patients in hospitals to determined average number of patients to be seen and the time each patient waits in the hospital before being attended to (Adeleke, Adebiyi, and Akinyemi, 2009).

The characteristics of a queuing model as asserted by Yasara (2009) are;

i) the population source
ii) the number of servers;
iii) the arrival patterns and the service patterns and
iv) the queue discipline.

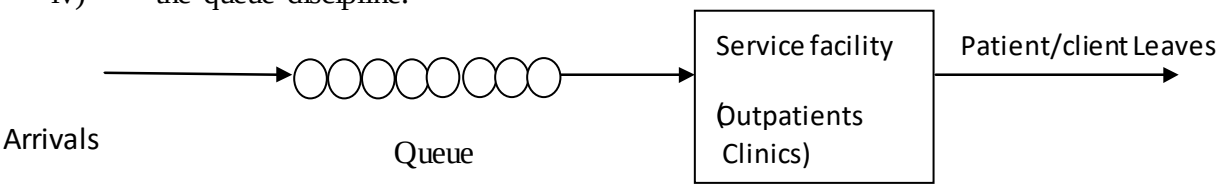

Fig. 1. Single line flow of patients in a clinic ("triage") and specialists at COPD.

According to Yasara (2009) Arrival patterns – the waiting lines occur because highly variable arrivals and service patterns cause the systems to be temporarily overloaded. The arrival pattern is different at different times of the day. Service patterns - because of the varying nature of the illnesses and the patients' conditions, the time required for treatment varies from patient to patient. Queue discipline refers to the order in which customers are processed. The assumption that service is provided on a first-come, first-served basis is the most commonly encountered rule. The Consultant Outpatient Department does not serve on this basis, patients do not all represent the same risk, level of triage; those with the highest risk, the most seriously ill, are treated first.

Thus, queuing can be applied to resolve the existing relations between service demand, number of doctors and the attention priority of the patient seen through a system of queues. To provide a good service, those responsible must use tools that allow them to analyze, program, plan, prioritize and, in general, decide on the best way to manage the available resources (Sinastava, Shenoy, & Sharma, 2008).

Abraham and Byrnes (2009) asserted that the theory can be used to analyzed and estimate the level of

service provided to the patients, the average waiting time, the number of patients queued, the capacity used, and the probability that the patient needs to wait. In hospital systems, the waiting time to receive attention is a key element in measuring the quality of the service and the waiting time of the patient.

To analyze a queue, the demand and the service times must be characterized. Once this information is obtained, the calculation of the properties is done. The method of least squares was used to verify the function that best suits the data of the arrivals and the services. This is important because in queueing theory several analytical models assume that the process follows a type of distribution and the related functions (Hall, 1991).

We focus on the exact drawing of two fundamental parameters from the digital data for queueing analysis, namely, the arrival rate (λ) and the service rate (μ), to overcome these problems. If we know the exact values of the arrival rate and the service rate, we can calculate the average waiting time in a queue (W_q). Thus, it is possible to measure the decrease in outpatients' waiting times. Almost all queueing models assume the probability distribution of the inter arrival time and the service time as an exponential distribution, and the number of arriving patients per unit of time follows Poisson distribution. Therefore, we assume that the probability distributions of the inter arrival and service times are exponential. Furthermore, because the purpose of this research was to analyze the change in outpatients' consultation time in terms of a hospital and as a result, the number of the server is one in this case, we use the queueing model of M/M/1 for the analysis.

From the assumption of queueing theory, patients who want to receive medical attention arrive randomly and wants the services to be provided immediately on arrival. In a normal situation, if the service facility is operating at a peak capacity when they arrive, they (patient) should wait in line. In this case, a queue is formed between the patient's arrival rate and the time required for services to be provided. Characteristically, the nature of medical services is such that one cannot predict when a patient will arrive and how much will be taken to provide the services. The ultimate objective of queueing theory is to achieve an economic equilibrium or a balance between the service cost and the patient's wasted time while waiting in the queue to be served. To achieve this a Measurement scale for a queueing system can be generated to include; the average number of customers in its queue (L_q), the average number of customers in its entire system including the entity being served (L_s), the average waiting time in the queue (W_q), and the average waiting time in its entire system (W_s) (Hall, 1991).

METHODS

The research design used for the study was descriptive design. This design was adopted for this study because it has the capacity to explore diverse population to assess the respondents' (patients) responses on the issue under investigation and in gathering data to measure the relationship between and among variables. Purposive sampling technique was adopted for the study. Since the outpatients services are provided in a centralized area with different clinics, simple random sampling was used to select the patients for the study.

We collected data from the out patients' department of the hospital using a questionnaire. A total of three hundred and fifty questionnaire were distributed to patients who were not critically ill for completion. Three hundred and twenty eight (328) copies of the questionnaire were properly completed and returned, representing 93.7% success rate. While 22 questionnaire were destroyed for want of information or improper completion. The data were extracted from the questionnaire and were used to analyze the research questions which results have been presented below.

DATA PRESENTATION AND ANALYSIS

Socio-demographic data of the respondents (n = 328)

Table 1: Socio-demographic data (n= 328)

VARIABLES	CATEGORY	FREQUENCY (N)	PERCENTAGES (%)
AGE	0 – 25 year	59	18
	26 – 30year	92	28
	35 – 40 years	46	14
	45 – 50 years	79	24
	51 – 55 years	32	10
	55+ years	20	6
	Total	328	100
RELIGION	Christian	210	64
	Muslim	30	9
	Traditional	88	27
	Total	328	100
EDUCATIONAL QUALIFIeCATION	Primary	52	16
	Secondary	146	45
	Tertiary	113	34
	Total	328	100
OCCUPATION	Farmer	13	4
	Trader	104	32
	Civil servant	155	47
	House wife	56	17
	Total	328	100
TRIBE	Ejagham	104	32
	Igbo	81	25
	Efik	56	17
	Others	87	26
	Total	328	100

Source: Fieldwork, 2021

Age: The socio-demographic data of the respondents indicates that 59(18%) were 0 – 25 years of age, 32(10%) were between 26 – 30years, 46(14%) were between 35 – 40 years, 79(24%) were between 45 – 50 years, 92(28%) were between 51 – 55 years and 20(6%) were 55years and above. The data revealed that respondents in the age bracket of 51 – 55 years dominated in the study.

Religion: The data revealed that, 298(91%) respondents were Christians, 30(9%) respondents were Muslims and 88 (27%) were African traditionalist. The study was carried out in a Christian dominated area.

Educational level: Out of the 328 respondents, 17(5%) had no form of education, 52(16%) had attained primary educational level, while 146(45%) had attained secondary level and 113(34%) had attained tertiary education.

Occupation: Majority of the respondents, 155(47%) were civil servants, 104(32%) were traders, 56(17%) were house wives, and 13(4%) were farmers.

Tribe: Most of the respondents, 104(32%) were Ejagham, 81(25%) Igbos 56(17%) Efiks and 87(26%) others tribes.

4. Results – Analysis of questions

Research question 1

Does Queuing of patients enhanced efficient service provision at the outpatient clinic of the Teaching Hospital? To answer this question, items 7-11 in Section B of the questionnaire were used.

Table 2: Queuing of patients enhanced efficient service provision

S/N	VARIABLES	RESPONSE				TOTAL	
		YES	%	NO	%	NO.	%
7	Do you know what is queuing for health services?	301	92	27	8	328	100
8	Do the hospital staff usually ask you to queue for services each time you come for treatment?	277	84	51	16	328	100
9	Does queueing for services enable you to be attended to faster than when there is no queue?	230	70	98	30	328	100
10	Does queueing for services increased efficiency of healthcare workers?	292	89	38	11	328	100
11	Does queueing ensures that health services are provided with ease?	260	79	68	21	328	100

Source: Fieldwork, 2021

Mean (X) = $\frac{1360}{5}$ = 272 $\frac{282}{5}$ = 56

Table 2 above showed the respondents responses on research question one (1), which was intended to assess the respondents' knowledge whether Queuing enhanced efficient service provision. In questionnaire item 7, 301(92%) respondents indicated that they know what is queuing for services, while 27(8%) respondents said they do not know. In questionnaire item 8, 277(84%) respondents accepted that they do queue for services each time they visit the hospital, while 51(16%) respondents rejected. In questionnaire item 9, 230 (70%) respondents accepted that queueing for services enable them to be attended to faster than when there is no queueing. While 98(30%) respondents said NO. In questionnaire item 10, 292(89%) respondents accepted that queueing for services increased efficiency of healthcare workers. While 38(11%) objected to the question. In questionnaire item 11, 260 (79%) respondents accepted that queueing ensures that health services are provided with ease, while 68(21%) said NO. The mean (x) scores for those who said YES and NO were established. Since the mean of those who said YES (272) is greater than the mean of those who said NO (56), it was concluded that Queuing of patients enhanced efficient service provision at the outpatient clinic of the Teaching Hospital.

4.3.2 Research question 2

Does application of queuing mathematical theory improve patients' waiting time at the outpatient clinics of the Teaching Hospital?

To answer this question, questionnaire items 12-15 in Section C of the questionnaire were used.

Table 3: Queuing mathematical theory and improvement in patients' waiting.

S/N	VARIABLES	RESPONSE				TOTAL	
		YES	%	NO	%	NO	%
12	Does queuing for service relates with the time a patient wait before being served?	310	95	18	5	328	100
13	Does queueing for services at the outpatient department of the hospital waste more time than when you do not queue?	259	79	69	21	328	100
14	Does queuing for services enable the hospital workers attain to patients than when patients are not asked to queue?	277	84	51	16	328	100
15	Does application of queuing mathematical theory improve patients' waiting time?	299	91	29	9	328	100

Source: Fieldwork, 2021

Mean (X) = $\frac{1145}{4}$ = 286 $\frac{167}{4}$ = 42

Table 3 showed the respondents' result on research question two. In questionnaire item 12, 310(95%) indicated that queuing for service relates with the time a patient wait before being served while 18(5%) indicated that it does not. In questionnaire item 13, 259(79%) respondents said queueing for services at the outpatient department of the hospital does not waste time than when there is no queue, while 69(21%) said queuing waste more time. In questionnaire item 14, 277(84%) respondents asserted that queuing for services enable the hospital workers attain to patients than when patients are not asked to queue, while 51(16%) said NO. In questionnaire item 15, 299(91%) affirmed that application of queuing mathematical theory improve patients' waiting time while 29(9%) said it does not.

The mean (x) scores for those who said YES and NO were established. Since the mean of those who said YES, 286 is greater than the mean of those who said NO, 42. It was concluded that Application of queuing mathematical theory improve patients' waiting time at the outpatient clinics of the Teaching Hospital.

Results for research questions 3

Does queuing mathematical theory enchance decongestion of patients at the outpatient clinics of the Teaching Hospital? To answer this question, items 16-19 on Section D of the questionnaire were used.

Table 4: queuing mathematical theory as a suitable model for managing congestion at the outpatient clinic**.**

S/N	VARIABLES	RESPONSE				TOTAL	
		YES	%	NO	%	NO	%
16	Can queuing theory be a suitable model for managing patients' congestion at the outpatients department?	322	98	6	2	328	100
17	Does queuing of patients enable hospital staff maintain orderliness at the outpatient department/ clinics?	323	98	5	2	328	100
18	Does application of queuing theory enhance decongestion of patients at the outpatient department/ clinics?	302	8	26	92	328	100
19	Does queuing of patients ensure uniform provision of services to patients without discrimination or bias?	303	92	25	8	328	100

Source: Fieldwork, 2021

$$\text{Mean (X)} = \frac{1250}{4} = 313 \qquad \frac{62}{4} = 16$$

Table 4 showed the result for research question three. In questionnaire item 16, 322(98%) out of the 328 respondents used for this study indicated that queuing theory is a suitable model for managing patients' congestion at the outpatients department, while 6(2%) indicated that it does not. In questionnaire item 17, 323(98%) said queuing of patients enable hospital staff to maintain orderliness at the outpatient department/ clinics, while 5(2%) said it does not. In questionnaire item 18, 302(92%) respondents agreed that queuing enable patients to exercise patience while waiting to be attained to by the hospital staff or workers, while 26(8%) did not assert to it. In questionnaire item 19, 303(92%) affirmed that queuing of patients ensure provision of services to patients without discrimination or bias, while 25(8%) objected.

The mean (x) scores for those who said YES and NO were established. Since the mean of those who said YES, 313 is greater than the mean of those who said NO, 16. It was concluded that queuing mathematical theory can serve as a suitable model for managing congestion at the outpatient clinic of the Teaching Hospital.

DISCUSSION

This study was carried out to examine the utilization of Queuing mathematical theory in the management of outpatient services in a Teaching Hospital, in South – South Geo political zone of Nigeria. The background of the study, research problem and the objective of the study were streamlined. The result of research question one (1) shows that Queuing of patients enhanced efficient service provision at the outpatient Department of the Teaching Hospital. This was in line with Olorunsola, Adeleke and Ogunlade (2014) assertion that, application of queueing theory to model health care is growing more popular as hospital management teams are becoming aware of the advantages it has in contributing to the quality of the service. That, improvement in health care services will not yield good results except the issue of time spent by patients at various hospital points is addressed. A successful and efficient health services delivery can only be achieved through a queuing discipline at various points where the services are provided.

The result of research question two revealed that application of queuing mathematical theory improve patients' waiting time at the outpatient clinics of the Teaching Hospital. This was in agreement with a study carried out by Sinastava, Shenoy, and Sharma (2008) which shows that long waiting time can affect patients by creating a low compliance with medical procedures and the doctor's recommendations, deterioration and dissatisfaction with their care and unwillingness to visit the hospitals even when seriously ill. To avoid such situations hospitals are increasingly challenged to reduce these inherent waits and delays in moving patients into and out of the hospital. Hence, establishing a queuing model for patients can reduce extensive waits and improve the overall flow of patients in the hospitals.

The responses in relation to research question 3 shows that queuing mathematical theory can serve as a suitable model for managing congestion at the outpatient clinic of the Teaching Hospital. The result was in accordance with the study conducted by Tochukwu, Obiora, and Okechukwu, (2017) on healthcare delays and the application of the discipline of queuing theory. It was discovered that congestion can be minimized by providing queuing models, and time-specific system to reduce overutilization of doctors at the hospitals at a minimum costs as against their present state. They further noted that health services require coordination of multiple resources, such as physicians, medications, and diagnostic equipment, services are provided in multiple steps, through a network of services, delays can be reduced through careful forecasting, scheduling, and process improvement and information management. The use of queuing models allows the capturing of the stochastic nature of arrivals and service time that is typical in health care systems.

RECOMMENDATIONS

The following recommendations were suggested for implementation;

i. Staff of the hospital should encourage patients to queue for the services as they arrived.
ii. Queuing system should be strictly followed and Patients should be attended base on queuing or first – come - first serve, except patients with critical problems.
iii. Multiple service points should be created enhance the effectiveness of the queues.

CONCLUSION

This study was carried out to examine effective utilization of Queuing mathematical theory in the effective management of outpatient services in a Teaching Hospital, in South – South Geo political zone of Nigeria. The result revealed that Queuing of patients enhanced efficient service provision at the outpatient Department of the Teaching Hospital; application of queuing mathematical theory improve patients' waiting time at the outpatient clinics (of the Teaching Hospital) and queuing mathematical theory can serve as a suitable model for managing congestion at the outpatient clinic of the Teaching Hospital. The study concluded that, queueing theory can be applied to improve

efficiency in the delivery of health care services at the outpatient department of the hospital where there is patients' congestion and service providers need to calculate the optimal supply of fixed resources necessary to meet a variable demand. The theory is a suitable tool that can be used to efficiently calculate some of the performance measures that could be of greater interest for the management of the hospital to control the hospital systems. These can in turn help to determine the cost of idle time and patients' waiting time.

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CONSENT AND ETHICAL APPROVAL

Approval was sought and granted by the institution. The purpose of the study was explained to each respondent and verbal consent was obtained from respondents prior to the time the questionnaire were to be administered. Confidentiality was ensured by not asking the respondents to write their names on the questionnaire and completed questionnaires were preserved to prevent unauthorized access by people who were not part of the research team.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

PUBLISHING RIGHTS

The authors have agreed to grant the editor of this journal approval for the first editing / publishing rights upon acceptance. We wish to state that this work has not been forwarded to another journal for consideration for publication.